Properties of Integer Exponents

Use What You Know

In the past, you have written and evaluated expressions with exponents such as 5^3 and $x^2 + 1$. Now, take a look at this problem.

Multiply: $(3^3)(3^4)$

Use the math you know to answer the questions.

a. What do the expressions (3^3) and (3^4) have in common? ______________________

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b. Write a multiplication expression without exponents that is equivalent to 3^3. ______________________

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c. How many factors of 3 did you write? ______________________

d. Write a multiplication expression without exponents that is equivalent to 3^4. ______________________

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e. How many factors of 3 did you write? ______________________

f. Write a multiplication expression without exponents that is equivalent to $(3^3)(3^4)$.

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g. How many factors of 3 did you write? ______________________

h. Write an expression with exponents to complete this equation: $(3^3)(3^4) =$ ______________________

i. What is the relationship between the exponents of the factors and the exponent of the product in your equation? ______________________

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j. Use words to explain how to multiply $(3^3)(3^4)$.

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Find Out More

You have seen one example of how to multiply powers with the same base. Here are two more:

$$(5^8)(5^5) = 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 = 5^{8+5} = 5^{13}$$

$$(x^6)(x^2) = x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x = x^{6+2} = x^8$$

In general, for the product of powers with the same base, $(n^a)(n^b) = n^{a+b}$, where $n \neq 0$.

You can also use the meaning of exponents to divide powers with the same base.

Divide $\frac{4^{12}}{4^5}$.

$$\frac{4^{12}}{4^5} = \frac{4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4}{4 \cdot 4 \cdot 4 \cdot 4 \cdot 4}$$

$$= \frac{4}{4} \cdot \frac{4}{4} \cdot \frac{4}{4} \cdot \frac{4}{4} \cdot \frac{4}{4} \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$$

$$= 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$$

$$= 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$$

$$= 4^7$$

4^{12} is twelve 4s multiplied together.
4^5 is five 4s multiplied together.
Any non-zero number divided by itself is 1.
Seven 4s multiplied together is 4^7.

So, $\frac{4^{12}}{4^5} = 4^7$. What is the relationship between the exponents of the dividend, divisor, and quotient? The exponent of the quotient is the exponent of the dividend minus the exponent of the divisor. $12 - 5 = 7$.

In general, for the quotient of two powers with the same base, $\frac{n^a}{n^b} = n^{a-b}$, where $n \neq 0$.

Reflect

1 Explain why $\frac{5^{10}}{5^2}$ equals 5^8.

Learn About Products of Powers

Read the problem below. Then explore how to find the product of powers with the same base *and* the same exponent.

Simplify: $(3^2)^4$

Model It You can write it another way.

$(3^2)^4 =$ means ***3 squared**, multiplied as a factor **4 times***.

$(3^2)^4 = 3^2 \cdot 3^2 \cdot 3^2 \cdot 3^2$

$(3^2)^4$ is the product of 4 powers, each with the same base (3) and the same exponent (2).

Solve It You can apply the associative property of multiplication.

$(3^2)^4 = 3^2 \cdot 3^2 \cdot 3^2 \cdot 3^2$	$(3^2)^4$ is the product of four 3^2s multiplied together.
$= (3^2 \cdot 3^2)\,(3^2 \cdot 3^2)$	Apply the associative property of multiplication.
$= (3^4)(3^4)$	This is the product of powers with the same base.
$= 3^{4+4}$	Add the exponents.
$= 3^8$	

Connect It **Now you will explore the concept from the previous page further.**

2 Simplify: $(3^2)^4 =$ ____________

3 Describe the relationship between the exponents of $(3^2)^4$ and the exponent of 3^8.

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4 Complete these examples of products of powers that have the same base and the same exponent.

$(5^8)^6 = 5^8 \cdot 5^8 \cdot 5^8 \cdot 5^8 \cdot 5^8 \cdot 5^8 = 5^{8+8+8+8+8+8} = 5^{8 \cdot 6} =$ ____________

$(953^7)^3 = 953^7 \cdot 953^7 \cdot 953^7 = 953^{7+7+7} = 953^{7 \cdot 3} =$ ____________

5 In general, for a product of powers that have the same base and the same exponent,

$(n^a)^b =$ ________, where $n \neq 0$.

Now look at how to simplify a product of powers when the bases are *different* and the exponents are the same.

Simplify: $(2^3)(4^3)$

6 Write an expression without exponents that is equivalent to $(2^3)(4^3)$. ____________

7 Apply the associative and commutative properties of multiplication to write your expression as the product of groups of $2 \cdot 4$. ____________

8 How many groups of $2 \cdot 4$ do you multiply together to get $(2^3)(4^3)$? ____________

9 Complete this equation: $(2^3)(4^3) = (2 \times 4)^{\square} = \square^3$

10 In general, for a product of powers that have different bases and the same exponent,

$(a^n)(b^n) =$ ________, where $a \neq 0$ and $b \neq 0$.

Try It **Use what you just learned to solve these problems. Write your answers using exponents. Show your work on a separate sheet of paper.**

11 Simplify: $(2^{18})^8 =$ ________

12 Simplify: $(4^9)(25^9) =$ ________, or ________

Learn About Zero and Negative Exponents

Read the problem below. Then explore simplifying expressions with exponents equal to zero.

Simplify: 5^0

Model It You can write it another way.

It doesn't make sense to ask yourself, "What is zero 5s multiplied together?" We will need to approach this problem another way.

So far, you have worked with powers where the exponents are counting numbers (1, 2, 3, . . .). The rules for working with powers are the same when the exponent is 0.

You have seen that when you multiply powers with bases that are the same you add the exponents.

$(5^0)(5^4) = 5^{0+4} = 5^4$

Solve It You can apply the identity property of multiplication.

You know that 1 times any expression is equivalent to that expression by the identity property of multiplication.

$(1)(5^4) = 5^4$

Because $(\mathbf{1})(5^{\mathbf{4}}) = 5^4$

and $(\mathbf{5^0})(5^{\mathbf{4}}) = 5^4$,

$\mathbf{5^0}$ must therefore be equal to **1**.

Connect It Now you will explore the concept from the previous page further.

13 Simplify: $5^0 =$ ___________

14 Complete these examples:

$12^0 =$ ___________

___________ $= 1$

$(-7)^0 =$ ___________

15 In general, for a power where the exponent is equal to 0, $n^0 =$ ___________, where $n \neq 0$.

The rules for products of powers also apply when the exponent is a negative integer.

16 Complete this equation: $(6^5)(6^{-5}) = 6^{\square} =$ ___________

17 You already know that a number times its reciprocal equals 1. For example, $3 \cdot \frac{1}{3} = \frac{3}{3} = 1$.

Now complete this equation: $6^5 \cdot \frac{1}{6^5} =$ ___________ $=$ ___________

18 Since $6^5 \cdot 6^{-5} =$ ___________ and $6^5 \cdot \frac{1}{6^5} =$ ___________, then $6^{-5} =$ ___________.

19 Complete these examples:

$10^{-6} =$ ___________

$(-34)^{-7} =$ ___________

___________ $= \frac{1}{142^{13}}$

20 In general, for a power where the exponent is a negative integer, $n^{-a} =$ ___________, where $n \neq 0$.

Try It Use what you just learned to solve these problems. Write your answers using positive exponents where appropriate. Show your work on a separate sheet of paper.

21 Simplify: $455^0 =$ ___________

22 Simplify: $19^{-4} =$ ___________

Practice Using Properties of Integer Exponents

Study the example below. Then solve problems 23–25.

Example

Simplify: $2^4 \cdot 2^{-7}$

Look at how you could show your work.

$2^4 \cdot 2^{-7}$	product of powers with equivalent bases
$= 2^{4+(-7)}$	add exponents
$= 2^{-3}$	power with a negative integer exponent
$= \frac{1}{2^3}$	reciprocal with positive exponent

Solution $2^4 \cdot 2^{-7} = \frac{1}{2^3}$

In this problem, you have to apply more than one rule of working with exponents.

Pair/Share

If x and a are counting numbers, is x^{-a} less than or greater than 1? Explain.

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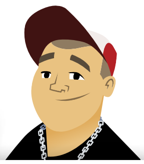
23 Simplify: $(3^2 \cdot 4^2)^5$

Show your work.



Remember the order of operations. Simplify the expression within the parentheses first.

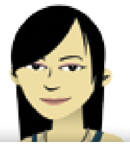
Solution ______________________

Pair/Share

Does $5^9 \cdot 6^7 = (30)^{16}$? Justify your answer.

24 Simplify: $9^{-8} \cdot \frac{1}{9^3}$. Write your answer with a positive exponent.

Show your work.



Remember what you know about adding negative numbers.

Pair/Share

Describe the value of the expression 0^{-5}.

Solution ______________________________

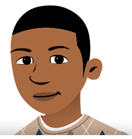
25 Which expression is equivalent to $\frac{45^{-3}}{45^3}$?

A 45^{-1}

B 45^0

C $\frac{1}{45^6}$

D 45^6



The expression is a quotient of powers.

Isaac chose **A** as the correct answer. How did he get that answer?

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Pair/Share

Talk about the problem and then write your answer together.

Practice Using Properties of Integer Exponents

Solve the problems.

1 Which expression is equivalent to $(-4^{-5})^0$?

A 1

B $(-4)^5$

C $\frac{1}{(-4)^5}$

D $\frac{1^5}{-4}$

2 Which expression is equivalent to $\frac{(7^2)^5}{7^{-6}}$?

A 7

B 7^4

C 7^{13}

D 7^{16}

3 Which expression is equivalent to $\frac{1}{49}$? Select all that apply.

A $7^{-1} \times 7^{-1}$

B $7^8 \times 7^{-6}$

C $7^{-5} \times 7^3$

D $7^7 \times 7^{-9}$

E $7^{-2} \times 7^4$

4 Write 16^8 as a power with a base of 4.

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5 Look at the equations below. Choose *True* or *False* for each equation.

a. $2^4 \times 3^4 = 4^6$ ☐ True ☐ False

b. $5^2 \div 5^3 = \frac{1}{5}$ ☐ True ☐ False

c. $(6^3)^4 = (6^4)^3$ ☐ True ☐ False

d. $\frac{3^2}{3^7} = 3^2 \times 3^{-7}$ ☐ True ☐ False

e. $\frac{8^0}{8^{-4}} = 8^{-4}$ ☐ True ☐ False

f. $4^{10} \div 4^5 = 4^2$ ☐ True ☐ False

6 Write each of these numbers as the product of a whole number and a power of 10. Then describe the relationship between place value and exponents.

3,000 = ______________________________

300 = ______________________________

30 = ______________________________

3 = ______________________________

0.3 = ______________________________

0.03 = ______________________________

0.003 = ______________________________

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✓ Self Check **Go back and see what you can check off on the Self Check on page 1.**